Multi-bit Decentralized Detection of a Non-cooperative Moving Target Through a Generalized Rao Test

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***Abstract*—We consider decentralized detection (DD) of an uncooperative moving target via wireless sensor networks (WSNs), measured in zero-mean unimodal noise. To address energy and bandwidth limitations, the sensors use multi-level quantizers. The encoded bits are then reported to a fusion center (FC) via binary symmetric channels. Herein, we propose a generalized Rao (G-Rao) test as a simpler alternative to the generalized likelihood ratio test (GLRT). Further, the asymptotic performance of a trajectory-clairvoyant (multi-bit) Rao test is leveraged to develop an offline and per-sensor quantizer design. Simulations show the appeal of G-Rao test with respect to the GLRT, and the gain in detection by using multiple bits for quantization.**

***Index Terms*—Decentralized detection, Generalized Rao test, GLRT, multibit quantizer, wireless sensor networks.**

I. Introduction

Wireless sensor networks (WSNs) have attracted significant interest due to their adoption in surveillance, environmental monitoring, smart cities, etc. Decentralized detection (DD) is one of the key tasks for a WSN; hence, scientific community has put significant effort in its study in last decades[1] [1]–[5].

Due to energy & bandwidth limitations, sensors are designed to *quantize* their measurements (into one or more bits), before sending them to a fusion center (FC) where a system-wide decision is taken [6], [7]. In this case, the optimal per-sensor design is a local likelihood-ratio (LR) quantization [8], [9], with corresponding selection of sensors' thresholds being a complex task [1], [2]. Thus the bit(s) sent either embodies the estimated binary event via a sub-optimal rule [10] (in one-bit case) or results from a "dumb" quantization [11].

In both cases sensors' bits are sent to the FC, where they are fused via an intelligently-designed rule meant to overcome sensors' limited detection capabilities. Sadly, the target to be detected depends on some unknown parameters. This precludes (global) LR implementation at FC [2], which is then faced to test a composite hypothesis. A commonly-adopted fusion rule in such cases corresponds to the generalized LR test (GLRT) [12]–[14]. Yet, in a case of an unknown static (resp. moving) target with unknown location (resp. trajectory), the GLRT requires a grid search on both the target location (trajectory) and emitted signal domains; this motivates the search for simpler fusion rules.

Hence, recent works have devised a *generalized* Rao test for one-bit DD of uncooperative targets in finite-sample [15] and sequential [16] setups. Sadly, there is useful information lost when only one-bit quantizers are adopted and a notable performance gap can be observed w.r.t. unquantized observations [17]. Accordingly, *multi-level quantization* can be adopted to achieve performance gains at the expenses of a mild complexity increase. Based on this idea, multi-bit DD has been recently considered for the simpler scenario of an unknown signal in Gaussian noise [18], [19], where *multi-bit* GLR (*not in closed-form*) and Rao tests (*in closed-form*) have been devised and an asymptotically-optimal thresholds' design obtained, via a particle swarm optimization (PSO) [20].

Accordingly, herein we study DD of a non-cooperative moving target via WSNs [11], [12], [18], with sensors using multi-bit quantizers. Our model encompasses: (i) *zero-mean unimodal-symmetric* noise pdfs; (ii) quantized data sent to the FC over error-prone links (emulating energy-limited communications) modeled as binary symmetric channels (BSCs). The resulting test is two-sided with nuisance parameters present only under hypothesis $\mathcal{H}_1$, thus making inapplicable the standard Rao test [21]. To circumvent this issue and capitalize multi-level measurements, we devise *a multi-bit form of generalized Rao test* (G-Rao), representing (i) a (computationally-) simpler alternative *fusion rule* to the GLRT and (ii) comprising the one-bit G-Rao devised in [15] as a special case. Also, we propose a quantizer design, based on asymptotic performance maximization of a *trajectory-clairvoyant* (TC) Rao test. Such design is per-sensor, accounts for sensor-FC channel status, and requires neither the target signal nor its trajectory, so it can be computed *offline* via PSO (following [18], [19]). Simulations compare both rules in a practical scenario[2].

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[2]*Notation* - vectors are denoted with lower-case bold letters, with a_n being the nth element of $\boldsymbol{a}$; finite sets are denoted with upper-case calligraphic letters, e.g. $\mathcal{A}$; transpose and expectation are denoted with $(\cdot)^T$ and $\mathbb{E}\{\cdot\}$, respectively; probability mass functions (pmfs) and probability density functions (pdfs) are denoted with $P(\cdot)$ and $p(\cdot)$, respectively, while $P(\cdot|\cdot)$ and $p(\cdot|\cdot)$ their corresponding conditional counterparts; the complementary cumulative distribution function (ccdf) is denoted with $F(\cdot)$; the symbols $\sim$ and $\overset{a}{\sim}$ mean "distributed as" and "asymptotically distributed as"; $\mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian PDF with mean μ and variance σ^2; χ_k^2 (resp. $\chi_k'^2(\xi)$) denotes a chi-square (resp. a non-central chi-square) pdf with k degrees of freedom (resp. and non-centrality parameter ξ).

II. Problem Statement

We consider a binary hypothesis test where a collection of sensors $k \in \mathcal{K} \triangleq \{1, \ldots, K\}$ are deployed to monitor the absence ($\mathcal{H}_0$) or presence ($\mathcal{H}_1$) of a target of interest having a partially-specified spatial signature. In the latter case (i.e. $\mathcal{H}_1$), the target moves along a fixed direction with constant velocity and continuously radiates an unknown deterministic isotropic signal θ. The emitted signal experiences distance-dependent path-loss and additive noise, before reaching individual sensors. The problem can be summarized as follows:

$$\begin{cases} \mathcal{H}_0 : & m_k^t = w_k^t, \\ \mathcal{H}_1 : & m_k^t = \theta\, g(\boldsymbol{x}^t, \boldsymbol{s}_k) + w_k^t \end{cases} ; \quad \begin{array}{c} k \in \mathcal{K}, \\ t = 1, \ldots, T \end{array} \tag{1}$$

In Eq. (1), $m_k^t \in \mathbb{R}$ denotes the kth sensor measurement at instant t and $w_k^t \in \mathbb{R}$ indicates the noise random variable (RV). The RVs w_k^t are assumed (a) statistically independent over space (sensors) and (b) i.i.d. over time. In detail we assume each noise RV has $\mathbb{E}\{w_k^t\} = 0$ and *unimodal symmetric* pdf[3] $p_{w_k}(\cdot)$. We underline that reliable estimation of the sensor noise pdf(s) can be achieved based on training data.

By denoting with $\boldsymbol{x}^0 \in \mathbb{R}^d$ and $\boldsymbol{v} \in \mathbb{R}^d$ the initial target location and the corresponding velocity, respectively, the target location at time t is given by the parametric expression $\boldsymbol{x}^t = \boldsymbol{x}^0 + \boldsymbol{v}t$. Herein, we make the reasonable assumption that *both* $\boldsymbol{x}^0$ and $\boldsymbol{v}$ are *unknown*. On the other hand, $\boldsymbol{s}_k \in \mathbb{R}^d$ denotes the *known* kth sensor position, as a result of a sensor self-localization procedure. The pair $(\boldsymbol{x}^t, \boldsymbol{s}_k)$ *uniquely* determines the value of $g(\boldsymbol{x}^t, \boldsymbol{s}_k)$, here denoting the amplitude attenuation function (AAF)[4], which models how the signal emitted from the target at t decays as it reaches kth sensor. For instance, when noise RVs are modelled as $w_k^t \sim \mathcal{N}(0, \sigma_{w,k}^2)$, the measurement m_k^t is distributed under $\mathcal{H}_0$ (resp. $\mathcal{H}_1$) as $m_k^t \,|\, \mathcal{H}_0 \sim \mathcal{N}(0, \sigma_{w,k}^2)$ (resp. $m_k^t \,|\, \mathcal{H}_1 \sim \mathcal{N}(\theta\, g(\boldsymbol{x}^t, \boldsymbol{s}_k),\, \sigma_{w,k}^2)$). For compactness, we define the set of unknowns as $\boldsymbol{\xi} \triangleq \{\theta, \boldsymbol{x}^0, \boldsymbol{v}\}$.

By looking at Eq. (1) we observe that the test is *two-sided*, namely $\{\mathcal{H}_0, \mathcal{H}_1\}$ corresponds to $\{\theta = \theta_0, \theta \neq \theta_0\}$ ($\theta_0 = 0$). More important, the *unknown* target position $\boldsymbol{x}^t$ (equivalently the nuisance parameters $\{\boldsymbol{x}^0, \boldsymbol{v}\}$) can be estimated at the FC *only* when $\theta \neq \theta_0$, i.e. when the signal is present ($\mathcal{H}_1$) [21].

Then, to address bandwidth & energy limited budget in WSNs, we assume that the kth sensor employs a (multi-level) $q(k)$-bit quantizer[5], in which the observation m_k^t is compared with a set of quantization thresholds $\{\tau_k(i)\}_{i=0}^{2^{q(k)}}$, determining $2^{q(k)}$ non-intersecting intervals covering the whole $\mathbb{R}$. Precisely, the corresponding quantizer outcome is mapped into a binary codeword $\boldsymbol{b}_k^t \in \{0,1\}^{q(k)}$, where $k = 1, 2, \ldots, K$. The quantization intervals are associated to $q(k)$-bit binary codewords, where $c_t(i) \in \{0, 1\}$ and $\boldsymbol{c}(i) = \begin{bmatrix} c_1(i) & \cdots & c_{q(k)}(i) \end{bmatrix}^T$. Hence, the $q(k)$-bit quantizer of kth sensor at instant t outputs a codeword defined as:

$$\boldsymbol{b}_k^t \triangleq \begin{cases} \boldsymbol{c}(1) & -\infty < m_k^t < \tau_k(1) \\ \boldsymbol{c}(2) & \tau_k(1) \leq m_k^t < \tau_k(2) \\ \vdots & \vdots \\ \boldsymbol{c}(2^{q(k)}) & \tau_k(2^{q(k)} - 1) \leq m_k^t < +\infty \end{cases} \tag{2}$$

The codeword of kth sensor is then reported to the FC via an error-prone channel link. The communication process of *each bit* is represented by an independent BSC with (known) bit-error probability (BEP) $P_{e,k}$. A *distorted* codeword $\boldsymbol{y}_k^t$ will be then received by the FC from kth sensor at time t, whose conditional probability obeys $P(\boldsymbol{y}_k^t = \boldsymbol{c}_k(i) | \boldsymbol{b}_k^t = \boldsymbol{c}_k(j)) = G_{q(k)}\left(P_{e,k}, d_{i,j}\right)$, where

$$G_{q(k)}\left(P_{e,k}, d_{i,j}\right) \triangleq P_{e,k}^{d_{i,j}} \left(1 - P_{e,k}\right)^{(q(k) - d_{i,j})}, \tag{3}$$

and $d_{i,j} \triangleq d(\boldsymbol{c}_k(i), \boldsymbol{c}_k(j))$ denotes the Hamming distance between codewords $\boldsymbol{c}_k(i)$ and $\boldsymbol{c}_k(j)$. For notation compactness, we collect the noisy codewords (the soft-quantized measurements) received from the sensors at time t in the set $\mathcal{Y}^t \triangleq \{\ \boldsymbol{y}_1^t \ \cdots \ \boldsymbol{y}_K^t\}$ (recall that $\boldsymbol{y}_k^t \in \{0,1\}^{q(k)}$ since codeword lengths may differ among sensors) and all the noisy codewords received from the WSN as $\mathcal{Y}^{1:T} \triangleq \{\mathcal{Y}^1, \cdots, \mathcal{Y}^T\}$. Accordingly, the pmf of all the observations, as a function of $\boldsymbol{\xi} = \{\theta, \boldsymbol{x}^0, \boldsymbol{v}\}$ is given by $p(\mathcal{Y}^{1:T}; \boldsymbol{\xi}) = \prod_{t=1}^T \prod_{k=1}^K P(\boldsymbol{y}_k^t; \boldsymbol{\xi})$. The corresponding pmf of the contribution from kth sensor at time t can be expanded as

$$P(\boldsymbol{y}_k^t; \boldsymbol{\xi}) = \sum_{i=1}^{2^{q(k)}} P(\boldsymbol{y}_k^t | \boldsymbol{b}_k^t = \boldsymbol{c}(i))\, P(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \boldsymbol{\xi}) \tag{4}$$

The quantizer law reported in Eq. (2) implies the following pmf expression for $P(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \boldsymbol{\xi})$

$$\begin{aligned} P(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \boldsymbol{\xi}) &= \Pr\{\tau_k(i-1) \leq m_k^t < \tau_k(i)\} = \\ F_{w_k}(\tau_k(i-1) - \theta\, g(\boldsymbol{x}^t, \boldsymbol{s}_k)) &- F_{w_k}(\tau_k(i) - \theta\, g(\boldsymbol{x}^t, \boldsymbol{s}_k)) \end{aligned} \tag{5}$$

where $F_{w_k}(\cdot)$ denotes the ccdf of w_k^t.

After receiving $\mathcal{Y}^{1:T}$, the FC takes a global decision. The aim of this paper is the derivation of a (computationally) simple test (based on the statistic $\Lambda(\mathcal{Y}^{1:T})$) deciding for $\mathcal{H}_1$ (resp. $\mathcal{H}_0$) when the statistic is above (resp. below) the threshold γ, and the design of quantizers for the *whole* WSN.

III. Fusion rules design

A widespread approach to handle composite hypothesis testing (viz. accounting for the presence of unknown parameters) resorts to the GLR [22]. For the DD problem at hand, the corresponding decision statistic is obtained by replacing the unknown parameters $\{\theta, \boldsymbol{x}^0, \boldsymbol{v}\}$ with their ML estimates $\{\hat{\theta}, \hat{\boldsymbol{x}}^0, \hat{\boldsymbol{v}}\}$ (under $\mathcal{H}_1$) in the log-LR, i.e.

$$\Lambda_{\text{G}} \triangleq \ln \frac{p(\mathcal{Y}^{1:T}; \hat{\theta}, \hat{\boldsymbol{x}}^0, \hat{\boldsymbol{v}})}{p(\mathcal{Y}^{1:T}; \theta_0)} = \sum_{t=1}^T \sum_{k=1}^K \ln \frac{P(\boldsymbol{y}_k^t; \hat{\theta}, \hat{\boldsymbol{x}}^0, \hat{\boldsymbol{v}})}{P(\boldsymbol{y}_k^t; \theta_0)} \tag{6}$$

[3] Noteworthy examples of such pdfs are the Gaussian, Laplace, Cauchy and generalized Gaussian distributions with zero mean [22].

[4] We underline that our results apply to any suitably-defined AAF describing the spatial signature of the target to be detected.

[5] Herein, for simplicity, we focus on deterministic quantizers, leaving the more general case of probabilistic quantizers [23] to future studies.

where $\theta_0 = 0$ and $\{\hat{\theta}, \hat{\boldsymbol{x}}^0, \hat{\boldsymbol{v}}\}$ are the maximum-likelihood estimates (MLEs), namely $\{\hat{\theta}, \hat{\boldsymbol{x}}^0, \hat{\boldsymbol{v}}\} \triangleq \arg\max_{\theta, \boldsymbol{x}^0, \boldsymbol{v}} p(\mathcal{Y}^{1:T}; \theta, \boldsymbol{x}^0, \boldsymbol{v})$. Note that the MLEs of such unknown parameters do not possess a closed-form. As a result, searching for the solution $\{\hat{\theta}, \hat{\boldsymbol{x}}^0, \hat{\boldsymbol{v}}\}$ may require a *huge computational burden*.

Hence, inspired by the approach in [15], [16], the G-Rao test statistic is considered here. The typical Rao test is known to be asymptotically equivalent to the GLRT *in weak-signal condition*[6], but with a lower computational complexity than the latter one. Referring to Eq. (1), if $\{\boldsymbol{x}_0, \boldsymbol{v}\}$ were known, the standard Rao test could be readily computed [22]. Sadly, the above terms are *unavailable* in our test. Still, leveraging Davies approach [21], a family of Rao test statistics can be calculated for different values of $(\boldsymbol{x}_0, \boldsymbol{v})$. Then by maximizing such family of statistics (following a "GLRT-like" approach), the G-Rao test statistic can be expressed as

$$\Lambda_{\mathrm{R}} \triangleq \max_{\{\boldsymbol{x}^0, \boldsymbol{v}\}} \left(\frac{\partial \ln \left[p(\mathcal{Y}^{1:T}; \boldsymbol{\xi}) \right]}{\partial \theta} \Big|_{\theta=\theta_0} \right)^2 / \, \mathrm{I}\left(\theta_0, \boldsymbol{x}^0, \boldsymbol{v}\right), \quad (7)$$

where $\mathrm{I}\left(\theta_0, \boldsymbol{x}^0, \boldsymbol{v}\right)$ denotes the *Fisher information* (FI) when $(\boldsymbol{x}_0, \boldsymbol{v})$ are known, i.e. $\mathrm{I}\left(\theta, \boldsymbol{x}^0, \boldsymbol{v}\right) \triangleq \mathbb{E}\left\{ \left(\frac{\partial \ln [p(\mathcal{Y}^{1:T}; \theta, \boldsymbol{x}^0, \boldsymbol{v})]}{\partial \theta} \right)^2 \right\}$, and evaluated at θ_0. Henceforth, we briefly describe the necessary steps for obtaining the G-Rao test in explicit form. First, we express the numerator term of Eq. (7) (before evaluating it at $\theta = \theta_0$) as

$$\left(\partial \ln \left[p(\mathcal{Y}^{1:T}; \boldsymbol{\xi}) \right] / \partial \theta \right)^2 = \quad (8)$$

$$\left(\sum_{t=1}^{T} \sum_{k=1}^{K} \frac{g\left(\boldsymbol{x}^t, \boldsymbol{s}_k\right) \sum_{i=1}^{2^{q(k)}} P(\boldsymbol{y}_k^t | \boldsymbol{b}_k^t = \boldsymbol{c}(i)) \, \rho(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \boldsymbol{\xi})}{\sum_{i=1}^{2^{q(k)}} P(\boldsymbol{y}_k^t | \boldsymbol{b}_k^t = \boldsymbol{c}(i)) \, P(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \boldsymbol{\xi})} \right)^2$$

where the auxiliary definition $\rho(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \boldsymbol{\xi}) \triangleq p_{w_k}\left(\tau_k(i-1) - \theta g(\boldsymbol{x}^t, \boldsymbol{s}_k)\right) - p_{w_k}\left(\tau_k(i) - \theta g(\boldsymbol{x}^t, \boldsymbol{s}_k)\right)$ has been employed. Secondly, we obtain the explicit form of the FI leveraging the result for multi-bit quantized measurements in [19] when replacing h_k with $g\left(\boldsymbol{x}^t, \boldsymbol{x}_k\right)$. This leads to $\mathrm{I}(\theta, \boldsymbol{x}^0, \boldsymbol{v}) = \sum_{k=1}^{K} \mathrm{i}_k(\theta, \boldsymbol{x}^0, \boldsymbol{v})$, where:

$$\mathrm{i}_k(\theta, \boldsymbol{x}^0, \boldsymbol{v}) \triangleq \sum_{t=1}^{T} g^2\left(\boldsymbol{x}^t, \boldsymbol{s}_k\right) \times \quad (9)$$

$$\sum_{i=1}^{2^{q(k)}} \frac{\left\{ \sum_{j=1}^{2^{q(k)}} G_{q(k)}\left(P_{e,k}, d_{i,j}\right) \rho\left(\boldsymbol{b}_k^t = \boldsymbol{c}\left(j\right); \boldsymbol{\xi}\right) \right\}^2}{\sum_{j=1}^{2^{q(k)}} G_{q(k)}\left(P_{e,k}, d_{i,j}\right) P\left(\boldsymbol{b}_k^t = \boldsymbol{c}\left(j\right); \boldsymbol{\xi}\right)}$$

[6] That is $|\theta_1 - \theta_0| = c/\sqrt{K}$ for some constant $c > 0$.

Last, we obtain the closed-form Λ_{R} by using both Eqs. (8)-(9):

$$\Lambda_{\mathrm{R}} = \max_{(\boldsymbol{x}^0, \boldsymbol{v})} \frac{1}{\mathrm{I}\left(\theta_0, \boldsymbol{x}^0, \boldsymbol{v}\right)} \times \quad (10)$$

$$\left(\sum_{t=1}^{T} \sum_{k=1}^{K} \frac{g\left(\boldsymbol{x}^t, \boldsymbol{s}_k\right) \sum_{i=1}^{2^{q(k)}} P(\boldsymbol{y}_k^t | \boldsymbol{b}_k^t = \boldsymbol{c}(i)) \, \rho(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \theta_0)}{\sum_{i=1}^{2^{q(k)}} P(\boldsymbol{y}_k^t | \boldsymbol{b}_k^t = \boldsymbol{c}(i)) \, P(\boldsymbol{b}_k^t = \boldsymbol{c}(i); \theta_0)} \right)^2$$

Despite the seemingly evaluation difficulty, Λ_{R} can be more easily evaluated than Eq. (6), since G-Rao only requires a grid search on the initial location $\boldsymbol{x}^0$ and velocity $\boldsymbol{v}$ (no need for estimating θ). Precisely, the *complexity* involved is $\mathcal{O}(N_{\boldsymbol{x}^0} N_{\boldsymbol{v}} \, T \sum_{k=1}^{K} 2^{q(k)})$ based on a 2-D grid, where $N_{\boldsymbol{x}^0}$ (resp. $N_{\boldsymbol{v}}$) is the number of initial position (resp. velocity) bins used. Differently, $\mathcal{O}(N_\theta \, N_{\boldsymbol{x}^0} N_{\boldsymbol{v}} \, T \sum_{k=1}^{K} 2^{q(k)})$ is required for a 3-D grid-based GLR, with an N_θ-fold saving for G-Rao.

IV. Quantizer design

The same quantizer design as [19] is precluded in our case, as no (asymptotically-)optimal performance expressions of tests based on the Davies approach are known in the literature [21]. Still, we modify the rationale in [19] to come up with a reasonable and feasible design. With this aim, we first consider the TC Rao statistic $\bar{\Lambda}_{\mathrm{R}}$, which has the knowledge of $(\boldsymbol{x}^0, \boldsymbol{v})$ and obeys the following asymptotic[7] pdf [19]:

$$\bar{\Lambda}_{\mathrm{R}} \overset{a}{\sim} \begin{cases} \chi_1^2 & \text{under} \quad \mathcal{H}_0 \\ \chi_1'^2(\lambda_q(\boldsymbol{x}^0, \boldsymbol{v})) & \text{under} \quad \mathcal{H}_1 \end{cases} \quad (11)$$

The above result also holds for the TC GLR. The term $\lambda_q(\boldsymbol{x}^0, \boldsymbol{v}) \triangleq \theta_1^2 \, \mathrm{I}(\theta_0, \boldsymbol{x}^0, \boldsymbol{v})$, ($\theta_1$ is the true value when $\mathcal{H}_1$ holds) represents the TC non-centrality parameter vs. $(\boldsymbol{x}^0, \boldsymbol{v})$ (the larger, the better the performance of TC GLR/Rao test when the known target trajectory is described by $(\boldsymbol{x}^0, \boldsymbol{v})$).

Evidently, the non-centrality parameter λ_q *grows monotonically* with the FI evaluated at θ_0. Such term depends on K quantization threshold vectors (one per sensor), with kth vector being $\boldsymbol{\tau}_k \triangleq \left[\tau_k(1), \ldots, \tau_k(2^{q(k)} - 1)\right]$, where the two extreme thresholds are fixed as $\tau_k(0) = -\infty$ and $\tau_k(2^{q(k)}) = +\infty$, respectively. Accordingly, we aim to optimize detection performance of the G-Rao detector (as well as GLRT) by solving the following maximization w.r.t. the $\boldsymbol{\tau}_k$'s:

$$\{\boldsymbol{\tau}_k^\star\}_{k=1}^K \triangleq \arg \max_{\{\boldsymbol{\tau}_k\}_{k=1}^K} \mathrm{I}\left(\theta_0, \boldsymbol{x}^0, \boldsymbol{v}, \{\boldsymbol{\tau}_k\}_{k=1}^K\right) \quad (12)$$

where the dependence of the FI on the $\boldsymbol{\tau}_k$'s has been highlighted, with a slight abuse of notation. In general, this (TC) procedure would lead to impractical $\boldsymbol{\tau}_k^\star$'s, namely depending on $(\boldsymbol{x}^0, \boldsymbol{v})$. Still, for our task, the objective admits a *decoupling* into a set of K separate threshold design problems. What's more, each problem is also *independent* of $(\boldsymbol{x}^0, \boldsymbol{v})$, namely:

$$\boldsymbol{\tau}_k^\star \triangleq \arg\max_{\boldsymbol{\tau}_k} \mathrm{i}_k(\theta_0; \boldsymbol{\tau}_k), \quad k = 1, \ldots, K \quad (13)$$

[7] Herein, the term "asymptotic" refers both to large-WSN and weak-signal conditions.

We highlight that each maximization is subject to the ordered constraints $\tau_k(i) < \tau_k(i+1)$, for $i = 1, \ldots 2^{q(k)} - 1$. Remarkably, the problem in (13) has the same form as [18], [19], resulting from optimization of Rao (GLR) test performance in a *simpler* unknown signal scenario. Thus, we adopt the method adopted therein, i.e. the PSO, to search each $\boldsymbol{\tau}_k^\star$ via (13), due to its appeal with high-dimensional and non-convex spaces[8].

Briefly, PSO is an optimization method [20] resorting to a swarm of $m = 1, 2, \ldots, M$ particles to explore[9] the $(2^q - 1)$-dimensional space (constrained in each dimension as $\tau(i) \in \{-\tau_{max}, \tau_{max}\}$) in search of a global maximum for each problem in Eq. (13). At ℓth iteration, the mth particle is characterized by its position $\boldsymbol{\tau}_m^\ell$ (i.e. the objective argument) and velocity vectors $\boldsymbol{\nu}_m^\ell$ (i.e. the improvement direction). The PSO state is summarized by the swarm best position ($\boldsymbol{sbest}^\ell$), as well as the best individual position achieved by mth particle so far ($\boldsymbol{pbest}_m^\ell$). At $(\ell+1)$th step, both terms contribute to update each particle velocity $\boldsymbol{\nu}_m^{\ell+1}$ and, indirectly, its position (via $\boldsymbol{\tau}_m^{\ell+1} = \boldsymbol{\tau}_m^\ell + \boldsymbol{\nu}_m^{\ell+1}$). PSO ends when all the particles' velocities (in magnitude) are less than a preset value $\leq \nu_{tol}$.

V. Numerical Results

Herein, we compare multi-bit G-Rao test and GLRT performance in terms of system false alarm ($P_{F_0} \triangleq \Pr\{\Lambda > \gamma | \mathcal{H}_0\}$) and detection probabilities ($P_{D_0} \triangleq \Pr\{\Lambda > \gamma | \mathcal{H}_1\}$). To this end, we consider a 2-D space ($\boldsymbol{x}_T \in \mathbb{R}^2$) where a WSN of size $K = 9$ is deployed to reveal the presence of an unknown moving target with its initial position located in the (square) surveillance area $\mathcal{L} \triangleq [0,1]^2$ and moving with velocity within $\mathcal{V} \triangleq [-0.1, 0.1]^2$. W.l.o.g., sensors are displaced in a regular grid covering $\mathcal{L}$. Concerning the sensing model, we assume $w_k \sim \mathcal{N}\left(0, \sigma_w^2\right)$, $k \in \mathcal{K}$, with $\sigma_w^2 = 1$. Also, we consider a power-law AAF $g\left(\boldsymbol{x}_T, \boldsymbol{s}_k\right) \triangleq 1/\sqrt{1 + (\|\boldsymbol{x}_T - \boldsymbol{s}_k\| / 0.2)^4}$. The target signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} \triangleq \theta^2/\sigma_w^2$. Herein, we set the *true* values as $\theta = 0.5$ ($\mathrm{SNR} = -3\,\mathrm{dB}$), $\boldsymbol{x}_0 = [0, 0.5]^T$, $\boldsymbol{v} = [0.02, 0.013]^T$ and $T = 20$ in the simulations to gain insight into detectors' performance.

Following Sec. III, Λ_{R} and Λ_{G} are evaluated by means of grids for $\boldsymbol{x}_0$, $\boldsymbol{v}$ and θ. Precisely, $\boldsymbol{x}_0$ and $\boldsymbol{v}$ are searched with $N_{\boldsymbol{x}^0} = N_{\boldsymbol{v}} = 100$ grid points uniformly sampling $\mathcal{L}$ and $\mathcal{V}$, respectively. Differently, the search space of θ (the target signal) is assumed to be $S_\theta \triangleq [-\bar{\theta}, \bar{\theta}]$ ($\bar{\theta} > 0$). The grid points[10] are then chosen as $\left[-\boldsymbol{g}_\theta^T \; 0 \; \boldsymbol{g}_\theta^T\right]^T$, where $\boldsymbol{g}_\theta$ collects target strengths corresponding to $\mathrm{SNR} = -10 : 1 : 10\,\mathrm{dB}$.

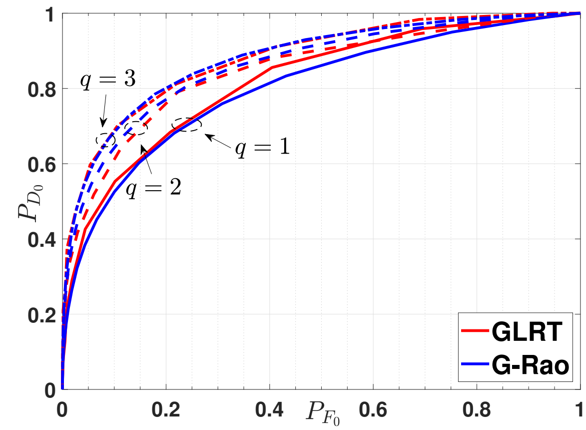
In Fig. 1 we show P_{D_0} vs. P_{F_0} of both G-Rao and GLRT detectors for (*a*) $P_{e,k} = P_e = 0$ and (*b*) $P_{e,k} = P_e = 0.1$, respectively (based on 10^5 Monte Carlo trials). For each case, we report the performance with $q_k = q \in \{1, 2, 3\}$ quantization bits, where thresholds are selected following the rationale elaborated in Sec. IV (via PSO, with parameters $M = 100$, $\tau_{max} = 5$ and $\nu_{tol} = 10^{-6}$). First, it is shown that the performance of the proposed G-Rao test is practically the same as the GLRT, while having the advantage of a lower computational burden with respect to the latter. Secondly, both fusion rules enjoy a higher detection probability (than the one-bit case) when using multi-bit quantizers.

Still, the presence of channel errors (in our example $P_e = 0.1$) leads to a significant performance loss of both detectors, highlighting the need for either a higher number of sensors (K) or a longer observation interval (T). A deeper numerical analysis will be brought out in the journal version of this paper.

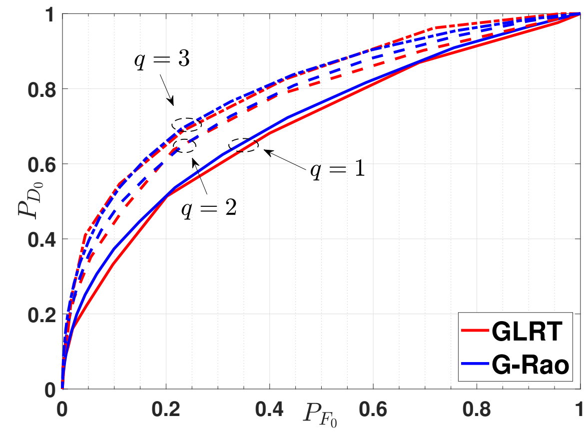
[8]We remark that other approaches may be pursued, e.g. simulated annealing.

[9]Hereinafter, aiming at improved notation readability, we will omit the sensor index subscript "k".

[10]This grid implies $N_\theta = 43$, hence a $43\times$ complexity saving is achieved by G-Rao w.r.t. GLR.



(a)



(b)

Fig. 1. P_{D_0} vs. P_{F_0} for GLRT and G-Rao; WSN with $K = 9$ sensors, $w_k \sim \mathcal{N}(0, 1)$, $\mathrm{SNR} = -3\,\mathrm{dB}$, and (a): $P_e = 0$; (b): $P_e = 0.1$.

VI. Concluding Remarks

We devised a G-Rao test for multi-bit DD of a non-cooperative moving target in WSNs. The considered model encompasses unimodal zero-mean symmetric noise, and non-identical BSCs. Our proposal constitutes a simpler alternative to the GLRT, while providing the same performance gains achieved via multi-bit quantization (over a one-bit counterpart). WSN performance was further optimized via the design of PSO-based quantizers, maximizing the asymptotic detection rate of TC Rao rule. Future avenues include the design of G-Rao test for sequential [16] and censoring [24] setups.

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